The Power of Mathematical Stories

Mathematics is full of stories. Stories that intrigue, like Fermat's note promising a proof too large to fit in the margins of the page, and stories of insight, like a naked Archimedes leaping from his bath shouting 'Eureka!'. When teaching, I often draw on stories to help bring mathematics to life. For example, legend has it that the French philosopher and mathematician René Descartes was lying in bed one morning, watching a fly crawling around on the ceiling above him. In thinking how to describe the fly's location, he realised that he could use one of the corners of the ceiling as a reference point and then describe the fly's position by using the distance from each wall to the fly. This method of locating a point in space by giving its relative distance from perpendicular intersecting lines is named after Descartes and you may know it as the *Cartesian coordinate system.*

Stories like these, which are probably more fiction than fact, add a richness to mathematics. However the stories that are of most interest to me as a mathematician and educator are the ones that help us make sense of mathematical ideas, where a narrative acts as the glue connecting isolated facts. Consider the facts of a well-known fable: three pigs, three houses made of varying quality of building materials, and a wolf. It's the narrative that is the connective tissue that turns these disparate facts into a story to remember.

I want to put <u>you</u> in a mathematical story now. Imagine you have a large pile of treasure. You also have two friends. You're a generous person, so you start by giving each of your friends a third of your treasure. But your friends want more. So you take the third you had kept for yourself, divide *it* into thirds, and share it out. Each of your friends now has 1/3 plus 1/9 of the treasure, and you only have 1/9. But now your friends are getting greedy. They are demanding more of the treasure, and you can't say no. So you give them each a third of what you have left. Each of your friends now has 1/3 plus 1/9 plus 1/27. And yet, they demand more! So you keep dividing whatever you have left into thirds and giving it to them, while keeping a third of what remains for yourself. This repeats until eventually you are left with nothing, and together they have all the treasure.

If you have been following along with this story, you can see that we can describe what each friend has as the sum of a series of fractions—1/3 + 1/9 + 1/27 and so on—and where each fraction in the series is a third of the previous one. However, when we look over at the treasure our friends have amassed in front of them, we can see that because each friend received the same amount each time, and we have nothing left, they each have exactly one-half of the initial pile of treasure.

This is a memorable story, with a narrative that hopefully captured your attention. And by thinking about this story in two different ways—the sum of the fractions and the total—we have made meaning of a sometimes-challenging mathematical idea. When I need to find the sum of an infinite geometric series, which is what this is, I don't try to remember the formula. Instead I conjure up the right number of greedy friends and a pile of treasure and tell myself a story.

Amie Albrecht

In his wonderful book 'The Mathematics of Human Flourishing', the American mathematician and college professor Francis Su writes that mathematical ideas "grow richer in meaning the more you play with them—each understanding brings a slightly different perspective—so that when you look at an idea in just the right way, you feel enlightened." When teaching, I try to be a mathematical storyteller—to help my students construct their own understanding by connecting seemingly disparate facts into a powerful collection of mathematical stories filled with meaning.

One way to do this is by experiencing mathematical concepts in a wide variety of ways. If I asked you to describe learning maths at school, you'll probably recall equation after equation. But symbolic descriptions are often the least natural way to begin to talk about mathematics. My colleagues and I frequently start with pictures, turning only to symbolic language when we need to be more precise about the details. Verbal descriptions, diagrams, physical objects, contextual situations—all bring a slightly different perspective.

One of our greatest mathematicians, Adelaide-born Terry Tao, tells an anecdote in which his aunt walked into her living room to find him rolling around on the floor with his eyes closed. He was trying to visualise a mathematical transform. He says: "I was pretending I was the thing being transformed. It did work actually, I got some intuition from doing that. Sometimes to understand something you just use whatever tools you have available."

Being able to move flexibly between representations deepens understanding. And, to use only one type is restrictive. It's like the parable of the blindfolded men encountering an elephant. Each man feels a different part of the elephant's body, such as the trunk or the tail or a tusk. When asked to describe what the creature looked like, they report very different animals based on their narrow experiences. Similarly, a mathematical representation can highlight only one aspect of a concept. A more complete picture emerges when one removes the blindfold and considers the idea from multiple perspectives.

Because stories can hold such power, we need to consider their effect. Without care, the mathematical stories we tell can reinforce bias and stereotypes. You may remember learning about Pascal's triangle at school. The 'triangle' in question is an arrangement of numbers. At the top of the triangle is a single 1. In the row below we write two 1s. The triangle grows row by row, with each entry of a row made by adding the two numbers above it. So the third row has three entries: 1 2 1. The next row has four entries: 1 3 3 1. The next row has five entries. And so on. In the West this is known as Pascal's triangle, named after French mathematician Blaise Pascal. However the same triangular arrangement of numbers is found in the texts of many other cultures, including Indian, Persian, and Chinese—hundreds of years before being published by Pascal. Many of us have encountered a predominantly Eurocentric view of mathematics, which is not always historically accurate.

In a similar way, the cast of characters of our mathematical stories is often incomplete. A challenge for you: how many women mathematicians can you name? In 2014, a survey found that a quarter of people in Europe were unable to name a single female scientist—living or dead. The t-shirt I am wearing is an Australian version of the MathGals project, designed to raise awareness of past and present women in mathematics. My shirt has the names of eight Australian women who should all be household names.

I will make special mention of Alison Doig Harcourt AO, now 92 and still active at The University of Melbourne. In the 1960s, Alison was part of a team that estimated, for the first time, the extent of poverty in Melbourne. In the 1980s she worked with a colleague to point out flaws on ballot papers, leading to changes in the methods used to order candidates in Australian elections. I learned of these achievements four years ago, when Alison was featured on the ABC's 7.30 program, the same year she was named Senior Victorian Australian of the Year. But what is particularly infuriating is that this media coverage was the first time I learned that Alison, along with British mathematician Ailsa Land, co-invented the 'branch and bound' algorithm, a fundamental technique in solving combinatorial optimisation problems that is taught in university classrooms around the world and was pivotal in my PhD research solving railway scheduling problems.

They say 'you can't be what you can't see'. There are many counterexamples to that—for a start, I'm one. But I do think it's easier to be what you can see. And that's why representation matters. Like my t-shirt. But representation also needs to go beyond increasing the visibility of women. For far too long Aboriginal mathematics and science has been devalued, denigrated, or simply ignored. We owe deep thanks to the tireless work of many for pushing back and making sure that these stories are being told.

And finally we come to perhaps the most important mathematical story—our own. How would you characterise yours? As a mystery? A foreign language? A romance? Or perhaps it feels more like a horror story. Let me reassure you that, whatever the genre, it is never too late to change your mathematical story. How can you look at it in a different way? Because when we are the authors of our own mathematical stories, that's powerful.